

The Physics Behind an Egg Drop: A Lively Story

Justin Paul Skycak, 2014

1. Velocity

Suppose you encounter a gigantic mountain troll on your way home one day. It starts walking in your direction... then it breaks into a run... and then a sprint. You're not quite sure what the troll's intentions are, but you make a prudent decision to flee. How fast must you run to escape the troll?

The answer to this question deals with the concept of **average velocity**, which is defined as:

$$\mathbf{v}_{\text{avg}} = \frac{\Delta \mathbf{x}}{\Delta t}$$

Where $\Delta \mathbf{x}$ is your change in position – the distance traveled, *in a specified direction* – and Δt is the amount of time needed for the given change in position

Velocity is very similar to speed – in fact, velocity IS speed, *in a specified direction*. Velocity is a directed quantity (also known as a vector); speed is a directionless quantity (also known as a scalar).

To escape the troll, you need to run faster than the troll. Thus, we can determine how fast you must run by finding the troll's velocity.

Ex. 1.1 Q: The troll chases you all the way to your school track. You notice that you are at the 100-meter-dash start line, so you decide to time how long it takes for the troll to run 100 meters. When the troll crosses the start line, you start counting, and when the troll crosses the finish line, you've counted 10 seconds. What is the troll's average velocity?

$$\mathbf{v}_{\text{avg,troll}} = \frac{\Delta \mathbf{x}_{\text{troll}}}{\Delta t} = (100 \text{ m in your direction}) / 10 \text{ s} = 10 \text{ m/s in your direction}$$

A: The troll has a velocity of 10 m/s in your direction.

Ex. 1.2 Q: You're not sure whether you're running fast enough, so you loop around the track so you can time yourself. This time, you start counting once you reach the 200-meter-dash start line, and you stop when you reach the finish line. It takes you 20 seconds. What is your velocity?

$$\mathbf{v}_{\text{avg,Ian}} = \frac{\Delta \mathbf{x}_{\text{Ian}}}{\Delta t} = (200 \text{ m in the direction of the track}) / 20 \text{ s} = 10 \text{ m/s in the direction of the track}$$

A: You have a velocity of 10 m/s in the direction of the track

Your top speed is the same as that of the troll. Fortunately, this means the troll isn't catching up to you. Unfortunately, you're not getting away.

Try this problem to check your understanding of velocity. The solution is on the last page of the document.

1.1) After being roundhouse kicked by batman, the joker flies through the air with a horizontal velocity of 20 m/s away from batman. He hits the ground after 3 seconds. How far away from batman does he land?

2. Momentum

We can agree that you and the troll have the same velocity. But why does the troll seem so much more powerful? It's because the troll has a lot of **momentum** in your direction. The troll's momentum is given by

$$\mathbf{p} = m\mathbf{v}$$

Where m is the **mass** of the troll and \mathbf{v} is the **velocity** of the troll.

Ex. 2.1 Q: As you run from the troll, you decide to estimate its momentum. If the mass of the troll is 2,000 kg (just under 4,500 lbm or 2.5 tons) and it is sprinting at a speed of 15 m/s (just over 30 mph), what is its momentum?

$$\begin{aligned}\mathbf{p}_{\text{troll}} &= m_{\text{troll}}\mathbf{v}_{\text{troll}} = 2,000 \text{ kg} * 15 \text{ m/s in your direction} \\ &= 30,000 \text{ kg}\cdot\text{m/s in your direction}\end{aligned}$$

A: The troll's momentum is 30,000 kg*m/s in your direction

No wonder you're running from the troll. If there was 30,000 kg*m/s coming in my direction, I would be running too.

Ex. 2.2 Q: You, too, are sprinting at a speed of 15 m/s. However, you weigh only 50 kg (just over 100 lbm). What is your momentum?

$$\begin{aligned}\mathbf{p}_{\text{Ian}} &= m_{\text{Ian}}\mathbf{v}_{\text{Ian}} = 50 \text{ kg} * 15 \text{ m/s in the direction of travel} \\ &= 750 \text{ kg}\cdot\text{m/s in the direction of travel}\end{aligned}$$

That's why you're not nearly as frightening as the troll.

Try these problems to check your understanding of momentum. The solutions are on the last page of the document.

2.1) What is the momentum of a 10^{-5} kg raindrop falling at a speed of 10 m/s?

2.2) A 1,500 kg car has a momentum of 45,000 kg*m/s to the east. What is its velocity?

2.3) A bird is flying through the sky with a speed of 12 m/s and has a momentum of 6 kg*m/s in the direction of flight. What is its mass?

3. Changes in momentum

The mountain troll has managed to keep up with you for ten minutes. Your legs begin to grow numb, and your heart is pounding so quickly you wonder if it might explode. You're running as fast as you can, but your top speed has decreased due to fatigue.

Along with a change in velocity, you have experienced a **change in momentum**. We denote your **change in momentum** by

$$\Delta \mathbf{p} = \mathbf{p}_{\text{final}} - \mathbf{p}_{\text{initial}}$$

Where $\mathbf{p}_{\text{final}}$ is your final (new) momentum and $\mathbf{p}_{\text{initial}}$ is your initial (old) momentum.

Ex. 3.1 Q: Your speed has decreased from 15 m/s to 10 m/s, but the direction of your velocity is the same. Assuming your mass of 50 kg has not changed, what is your change in momentum?

$$\begin{aligned}\Delta \mathbf{p}_{\text{Ian}} &= \mathbf{p}_{\text{Ian,final}} - \mathbf{p}_{\text{Ian,initial}} \\ &= 50 \text{ kg} * 10 \text{ m/s} - 50 \text{ kg} * 15 \text{ m/s in direction of travel} \\ &= 50 \text{ kg} * (-5) \text{ m/s in direction of travel} \\ &= -250 \text{ kg*m/s in direction of travel}\end{aligned}$$

A: Your change in momentum is -250 kg*m/s in your direction of travel. In other words, your momentum has decreased by 250 kg*m/s in your direction of travel.

Luckily, the troll is getting tired too.

Ex. 3.2 Q: The troll's speed has also decreased from 15 m/s to 10 m/s (but it is still chasing you). Assuming its mass of 2,000 kg has not changed, what is its change in momentum?

$$\begin{aligned}\Delta \mathbf{p}_{\text{troll}} &= \mathbf{p}_{\text{troll,f}} - \mathbf{p}_{\text{troll,i}} \\ &= 2,000 \text{ kg} * 10 \text{ m/s} - 2,000 \text{ kg} * 15 \text{ m/s in direction of travel} \\ &= 2,000 \text{ kg} * (-5) \text{ m/s in direction of travel} \\ &= -10,000 \text{ kg*m/s in direction of travel}\end{aligned}$$

A: The troll's change in momentum is -10,000 kg*m/s in your direction. In other words, the troll's momentum has decreased by 10,000 kg*m/s in your direction.

Keep in mind that a change in speed is not the only cause of a change in momentum. In total, there are three situations that may give rise to change in momentum:

- Change in *magnitude* of velocity (change in speed)
- Change in *direction* of velocity
- Change in mass

Try these problems to check your understanding of changes in momentum. The solutions are on the last page of the document.

3.1) A skydiver jumps out of a plane and reaches terminal velocity (maximum velocity due to air resistance) of 15 m/s toward the ground. After releasing his parachute, the skydiver has a new velocity of 3 m/s toward the ground. Assuming the skydiver and his equipment together weigh 100 kg, what is the skydiver's change in momentum?

3.2) A bird flies into the skydiver's parachute and rips a hole in the center. The skydiver's change in momentum is 500 kg*m/s toward the ground, taking initial momentum as immediately before the bird incident. What is the skydiver's new velocity?

4. Force

Like momentum, force is one of the key concepts at the heart of physics. And at the heart of Star Wars. When Yoda uses *the Force* to retrieve his lightsaber, he does so by imparting a *force* on it. This new force concept might help you escape that mountain troll, too, so let's take a closer look.

The best understanding of **force** is achieved not by *explicitly* defining it, but by *implicitly* defining it in terms of a change in momentum:

$$\mathbf{F}_{avg}\Delta t = \Delta\mathbf{p}$$

Where \mathbf{F}_{avg} is **average force** and Δt is a change in time. That's right – when a force acts upon an object for a period of time, the result is a change in momentum.

Thus, an (almost) equivalent expression of force is

$$\mathbf{F}_{avg} = \frac{\Delta\mathbf{p}}{\Delta t} ,$$

the change in momentum divided by the change in time.

Ex. 4.1 Q: Yoda uses the Force to pick up his 1 kg lightsaber, which is initially at rest. Just before he catches it, the lightsaber is traveling with a velocity of 10 m/s toward him. If this feat takes Yoda 2 seconds to accomplish, what average force did Yoda use on the lightsaber?

$$\begin{aligned}\mathbf{F}_{avg} &= \frac{\Delta\mathbf{p}}{\Delta t} = (1 \text{ kg} * 10 \text{ m/s toward Yoda} - 1 \text{ kg} * 0 \text{ m/s toward Yoda}) / 2 \text{ s} \\ &= (10 \text{ kg}\cdot\text{m/s toward Yoda}) / 2 \text{ s} \\ &= 5 \text{ kg}\cdot\text{m/s}^2 \text{ toward Yoda}\end{aligned}$$

A: Yoda used an average force of 5 kg*m/s² toward himself.

Wow. A kilogram-meter per second-squared (kg*m/s²) is a mouthful. Fortunately, no physicist says that because there's another name for it: a Newton (N). Looking back at Ex. 3.1, a more concise (yet equivalent) answer is that Yoda used an average force of 5 N toward himself.

Try this problem to check your understanding of force. The solution is on the last page of the document.

4.1) Yoda then throws his lightsaber by imparting a force (on the lightsaber) of 10 N for half a second toward Count Dooku. Assuming Yoda's lightsaber has a mass of 1 kg, what is its velocity when it hits Count Dooku

5. Pressure

Talking about Star Wars is great, but right now we have more important things to deal with – namely, a mountain troll that won't stop chasing you.

In hopes of deterring the troll, you throw a rock at it. Perhaps, you think, if you throw it hard enough, you can break one of the troll's bones, and it won't be able to run any more. But how hard do you have to throw it?

To determine how fast the rock must travel before it collides with the troll, we first need to understand **pressure**.

Pressure is defined as

$$P = F/A$$

where F is the magnitude of a force acting on an area A.

Often, there is a threshold to how much pressure an object can tolerate. For example, the average human femur can tolerate up to 160 ppsi (pounds per square inch), roughly $1.1 * 10^6$ N/m².

Ex. 5.1 Q: Suppose the troll's femur can tolerate $4 * 10^6$ N/m² of pressure, roughly four times as much as a human's. You pick up a 1 kg rock and throw it with a velocity of 30 m/s (roughly 70 mph) toward the troll. If the rock bounces off the troll's leg in a tenth of a second, over an area of 0.001 m² (10 cm²), does it break the troll's leg?

Let's first find the force that acts on the rock in the collision:

$$\begin{aligned} \mathbf{F}_{\text{avg}} &= \frac{\Delta \mathbf{p}}{\Delta t} = (1 \text{ kg} * 0 \text{ m/s toward the troll} - 1 \text{ kg} * 30 \text{ m/s toward the troll}) / 0.1 \text{ s} \\ &= (-30 \text{ kg*m/s toward the troll}) / 0.1 \text{ s} \\ &= -300 \text{ kg*m/s toward the troll} \\ &= -300 \text{ N toward the troll} \\ &= 300 \text{ N away from the troll} \end{aligned}$$

The troll's leg supplies this force.

Have you ever heard the phrase "for every action, there is an equal and opposite reaction? That's a summary of Newton's third law, and it can be used here to find the force on the troll's leg: since the troll's leg exerts 300 N (away from the troll) on the rock, the rock exerts 300 N (toward the troll) on the troll's leg.

Now that we know the force on the troll's leg, let's find the pressure.

$$P = F/A = 300 \text{ N} / 0.001 \text{ m}^2 = 300,000 \text{ N/m}^2 = 3 * 10^5 \text{ N/m}^2$$

However, this is less than the $4 * 10^6 \text{ N/m}^2$ breaking point

A: Unfortunately, the rock does not break the troll's leg.

Try these problems to check your understanding of pressure. The solutions are on the last page of the document.

5.1) Suppose you're trying to carve a block of wood with a dull knife. You exert 3 N of force on the knife, which makes contact with the block over an area of 0.05 cm^2 . How much pressure is exerted on the block? You can leave your answer in terms of N/cm^2 .

5.2) Realizing that your knife is not carving very well, you decide to sharpen it. After the knife is sharpened, the contact area becomes 0.001 cm^2 . If you again exert a force of 3N on the knife, how much pressure is exerted on the block? You can leave your answer in terms of N/cm^2 .

6. Troll Egg Drop

In the previous section, we realized that throwing a rock will not result in enough pressure to break the troll's femur. But what if you found a way to use the troll's own weight against itself? We all know what happens to an egg that is dropped from the roof of a house. It falls, and once it hits the ground, it breaks. Would the same thing happen to a troll if you somehow got it to jump off a ledge? Using the concepts from all the previous sections, and a few other tricks, we can find out.

Remember, we're looking for the pressure exerted on the troll when it hits the ground. Also, we're going to assume that all the pressure in the collision is exerted on the troll's femur. Although this is not realistic, it greatly simplifies the calculations.

We know from section 4 that pressure is given by $P = F/A$, so if we want to find the pressure we have to find the force exerted on the troll when it hits the ground and the area over which it contacts the ground during the collision. Finding the area is easy – it's just twice the area of a troll foot. We can approximate it later. However, finding the force will take a little more brainpower.

Well, we know from section 3 that $F_{avg} = \Delta p / \Delta t$, and we know from sections 1 and 2 that $\Delta p = m\mathbf{v}_{final} - m\mathbf{v}_{initial}$, so we can say that the force is

$$F_{avg} = (m\mathbf{v}_{final} - m\mathbf{v}_{initial}) / \Delta t.$$

The final velocity, the velocity of the troll just after it hits the ground, must be 0. Thus, we have

$$F_{avg} = -m\mathbf{v}_{initial} / \Delta t$$

Now, we have to be careful. The initial velocity refers to the velocity of the troll just before the collision – NOT the velocity of the troll when it jumps off the ledge. After the troll jumps off the ledge, it falls – its velocity toward the ground increases. We need to find the velocity of the troll after its velocity has increased from falling, but before it collides with the ground.

Unfortunately, you can't find this velocity using the methods you've learned thus far. Fortunately, there's an easy way to calculate it.

Using more advanced math and the specific constraints of this situation, we could derive the equation

$$mgh_{initial} = mv_{final}^2 / 2$$

for an object that falls from a height h_{initial} . The term on the left is an approximation of the initial gravitational potential energy of the object, and the term on the right is the final kinetic energy of the object. g denotes the acceleration due to gravity, $g = 9.81 \text{ m/s}^2$

We won't worry about how this equation was derived and why it works in this situation, but it's important that you realize this equation can be easily reasoned using concepts from calculus. It is NOT something I pulled out of thin air. This equation, along with nearly every equation in physics, is simply the result of logic.

Solving for v_{final} , we find

$$v_{\text{final}} = \sqrt{2gh_{\text{initial}}}$$

For the falling object. Remember, in this case v_{final} represents the velocity of the object just before the object collides with the ground. This is exactly our v_{initial} when we are referring to the collision!

Furthermore, we know that v_{final} is directed toward the ground. Thus, we can write it as a vector:

$$\mathbf{v}_{\text{final}} = \sqrt{2gh_{\text{initial}}} \text{ toward the ground}$$

Now that we know that $\mathbf{v}_{\text{initial}}$ in our collision problem is the same as $\mathbf{v}_{\text{final}}$ in the falling problem, we can say

$$\mathbf{F}_{\text{avg}} = -\frac{m\sqrt{2gh_{\text{initial}}}}{\Delta t} \text{ toward the ground}$$

And we can remove the negative sign by saying

$$\mathbf{F}_{\text{avg}} = \frac{m\sqrt{2gh_{\text{initial}}}}{\Delta t} \text{ away from the ground}$$

Or, more clearly,

$$\mathbf{F}_{\text{avg}} = \frac{m\sqrt{2gh_{\text{initial}}}}{\Delta t} \text{ toward the troll}$$

Now we can put this back into our equation for pressure, $P = F/A$, to get

$$P = \frac{m\sqrt{2gh_{\text{initial}}}}{A\Delta t}$$

Well, isn't that neat! We now have an equation for the pressure exerted on an object of mass m that falls from a height h_{initial} and collides with the ground over an area A and a time interval Δt ! Now we can decide whether leading the troll off a ledge will cause it to break its femur.

Ex. 6.1 Q: Suppose the 2,000 kg troll jumps off a 3 m ledge and hits the grass below over a period of 0.1 s. If each of the troll's feet covers 0.1 m^2 , will the troll break its femur? Remember, a troll femur will snap under a pressure greater than $4 * 10^6 \text{ N/m}^2$.

$$\begin{aligned}
 P &= \frac{m\sqrt{2gh_{\text{initial}}}}{A\Delta t} \\
 &= \frac{2000 \text{ kg} * \sqrt{2 * 9.81 \text{ m/s}^2 * 3 \text{ m}}}{2 * 0.1 \text{ m}^2 * 0.1 \text{ s}} \\
 &= \frac{2000 \text{ kg} * \sqrt{58.86 \text{ m}^2/\text{s}^2}}{0.2 \text{ m}^2 * \text{s}} \\
 &= 100000 \frac{\text{kg}}{\text{m}^2 * \text{s}} * \sqrt{58.86} \frac{\text{m}}{\text{s}} \\
 &\approx 100000 \frac{\text{kg}}{\text{m} * \text{s}^2} * 7.67 \\
 &= 767000 \text{ N/m}^2 \\
 &\approx 0.8 * 10^6 \text{ N/m}^2
 \end{aligned}$$

A: Unfortunately, the troll will experience a pressure of only $0.8 * 10^6 \text{ N/m}^2$, which is less than the breaking point $4 * 10^6 \text{ N/m}^2$. The troll's femur will not break

Well... This isn't good. There's no way you're going to lose that troll, huh?

Actually...

We have one last trick. Remember, the pressure on the troll is given by $P = \frac{m\sqrt{2gh_{\text{initial}}}}{A\Delta t}$, so we can increase the pressure P by

- *Increasing the mass m or*
- *Increasing the height h_{initial} or*
- *Decreasing the area A or*
- *Decreasing the time interval Δt*

In this situation...

- *Can we increase the troll's mass? No.*

- *Can we increase the height of the ledge?* Perhaps. We could try to find a higher ledge, but the likelihood of a troll jumping off a ledge taller than itself is not too favorable. Let's keep looking for other options.
- *Can we decrease the area of the troll's feet?* No.
- *Can we decrease the time of collision?* Definitely! In the last example, we assumed that the troll was landing on grass. If the troll lands on a harder surface, such as concrete, the collision time will decrease. There are plenty of ledges with concrete at the bottom.

Okay, last shot – let's see what happens when the troll lands on concrete.

Ex. 6.2 Q: Suppose the 2,000 kg troll jumps off a 3 m ledge and hits the **concrete** below over a period of **0.005 s** (20 times shorter than for grass). If each of the troll's feet covers 0.1 m^2 , will the troll break its femur? Remember, a troll femur will snap under a pressure greater than $4 * 10^6 \text{ N/m}^2$.

$$\begin{aligned}
 p &= \frac{m\sqrt{2gh_{\text{initial}}}}{A\Delta t} \\
 &= \frac{2000 \text{ kg} * \sqrt{2 * 9.81 \text{ m/s}^2 * 3 \text{ m}}}{2 * 0.1 \text{ m}^2 * 0.005 \text{ s}} \\
 &= \frac{2000 \text{ kg} * \sqrt{58.86 \text{ m}^2/\text{s}^2}}{0.001 \text{ m}^2 * \text{s}} \\
 &= 2000000 \frac{\text{kg}}{\text{m}^2 * \text{s}} * \sqrt{58.86} \frac{\text{m}}{\text{s}} \\
 &\approx 2000000 \frac{\text{kg}}{\text{m} * \text{s}^2} * 7.67 \\
 &= 15340000 \text{ N/m}^2 \\
 &\approx 15.3 * 10^6 \text{ N/m}^2
 \end{aligned}$$

A: Yes! The troll will experience a pressure of $15.3 * 10^6 \text{ N/m}^2$, which is greater than the breaking point $4 * 10^6 \text{ N/m}^2$. The troll's femur will break!

Great! Now that we know the troll's femur will break, we just have to find a way to lead it over the ledge.

What if you jump first, so the troll has to jump in order to follow you? Yes, your feet have a smaller area your femur is easier to break than the troll's. But your mass is also much smaller than the troll's. Let's carry out the calculation and see what happens.

Ex. 6.3 Q: Suppose you jump off a 3 m ledge and hit the concrete below over a period of 0.005 s. If each of your feet covers 0.01 m^2 , and you have a mass of 50 kg, will

you break your femur? Remember, your femur will snap under a pressure greater than $1.6 * 10^6 \text{ N/m}^2$.

$$\begin{aligned}
 p &= \frac{m\sqrt{2gh_{\text{initial}}}}{A\Delta t} \\
 &= \frac{50 \text{ kg} * \sqrt{2 * 9.81 \text{ m/s}^2 * 3 \text{ m}}}{2 * 0.01 \text{ m}^2 * 0.005 \text{ s}} \\
 &= \frac{50 \text{ kg} * \sqrt{58.86 \text{ m}^2/\text{s}^2}}{0.0001 \text{ m}^2 * \text{s}} \\
 &= 500000 \frac{\text{kg}}{\text{m}^2 * \text{s}} * \sqrt{58.86} \frac{\text{m}}{\text{s}} \\
 &\approx 500000 \frac{\text{kg}}{\text{m} * \text{s}^2} * 7.67 \\
 &= 3835000 \text{ N/m}^2 \\
 &\approx 3.8 * 10^6 \text{ N/m}^2
 \end{aligned}$$

A: Yeah... $3.8 * 10^6 \text{ N/m}^2$ is greater than your femur's breaking point of $1.6 * 10^6 \text{ N/m}$. Your femur will break too.

The whole point of breaking the troll's femur is so you can run away from it. If you have a broken femur as well, you can't run.

However, there is you can decrease the pressure exerted on you during the collision. Unlike the troll, whose tendons have the elasticity of rubber bands dipped in liquid nitrogen (i.e. very inelastic), you can increase your time of collision if you bend your knees as you collide with the concrete. Let's rework that last example with a greater collision time, since you are going to bend your knees to increase your time of collision.

Ex. 6.4 Q: Suppose you jump off a 3 m ledge and hit the concrete below over a period of 0.05 s (10 times longer than the collision time that assumes no knee-bending). If each of your feet covers 0.01 m^2 , and you have a mass of 50 kg, will you break your femur? Remember, your femur will snap under a pressure greater than $1.6 * 10^6 \text{ N/m}^2$.

$$\begin{aligned}
 p &= \frac{m\sqrt{2gh_{\text{initial}}}}{A\Delta t} \\
 &= \frac{50 \text{ kg} * \sqrt{2 * 9.81 \text{ m/s}^2 * 3 \text{ m}}}{2 * 0.01 \text{ m}^2 * 0.05 \text{ s}} \\
 &= \frac{50 \text{ kg} * \sqrt{58.86 \text{ m}^2/\text{s}^2}}{0.001 \text{ m}^2 * \text{s}} \\
 &= 50000 \frac{\text{kg}}{\text{m}^2 * \text{s}} * \sqrt{58.86} \frac{\text{m}}{\text{s}} \\
 &\approx 50000 \frac{\text{kg}}{\text{m} * \text{s}^2} * 7.67 \\
 &= 383500 \text{ N/m}^2 \\
 &\approx 0.38 * 10^6 \text{ N/m}^2
 \end{aligned}$$

A: $0.38 * 10^6 \text{ N/m}^2$ is smaller than your femur's breaking point of $1.6 * 10^6 \text{ N/m}^2$!
Your femur won't break!

Congratulations, you've outsmarted the troll. Our work is done.

Surprise! No problems. There were many examples in this section – if you can work all the examples yourself without looking at the solution, you probably have a decent understanding of the material.

Some Solutions

$$\begin{aligned}0.1) \mathbf{v}_{avg} &= \frac{\Delta \mathbf{x}}{\Delta t} \Rightarrow \Delta \mathbf{x} = \mathbf{v}_{avg} \Delta t \\ &= 20 \text{ m/s away from batman} * 3 \text{ s} \\ &= 60 \text{ m away from batman}\end{aligned}$$

$$\begin{aligned}1.1) \mathbf{p} &= m\mathbf{v} = 10^{-5} \text{ kg} * 10 \text{ m/s toward the ground} \\ &= \underline{10^{-4} \text{ kg*m/s toward the ground}}\end{aligned}$$

$$\begin{aligned}1.2) \mathbf{p} &= m\mathbf{v} \Rightarrow \mathbf{v} = \mathbf{p}/m = \frac{45,000 \text{ kg*m/s to the east}}{1,500 \text{ kg}} \\ &= \underline{30 \text{ m/s to the east}}\end{aligned}$$

$$\begin{aligned}1.3) \mathbf{p} &= m\mathbf{v} \Rightarrow m = p/v = \frac{6 \text{ kg*m/s}}{12 \text{ m/s}} \\ &= \underline{0.5 \text{ kg}}\end{aligned}$$

$$\begin{aligned}2.1) \Delta \mathbf{p} &= \mathbf{p}_f - \mathbf{p}_i = 3 \text{ m/s} * 100 \text{ kg} - 15 \text{ m/s} * 100 \text{ kg} \\ &= (-12) \text{ m/s} * 100 \text{ kg} \\ &= \underline{-1200 \text{ kg*m/s}}\end{aligned}$$

$$\begin{aligned}2.2) \Delta \mathbf{p} &= \mathbf{p}_f - \mathbf{p}_i \Rightarrow \mathbf{p}_f = \Delta \mathbf{p} + \mathbf{p}_i \Rightarrow \mathbf{v}_f = (\Delta \mathbf{p} + \mathbf{p}_i)/m \\ &= (500 \text{ kg*m/s} + 100 \text{ kg} * 3 \text{ m/s}) / 100 \text{ kg toward ground} \\ &= (800 \text{ kg} * \text{m/s}) / 100 \text{ kg toward ground} \\ &= \underline{8 \text{ m/s toward ground}}\end{aligned}$$

$$\begin{aligned}3.1) \mathbf{F}_{avg} \Delta t &= \Delta \mathbf{p} = \mathbf{p}_f - \mathbf{p}_i \Rightarrow \mathbf{p}_f = \mathbf{F}_{avg} \Delta t + \mathbf{p}_i \\ &\Rightarrow \mathbf{v}_f = (\mathbf{F}_{avg} \Delta t + \mathbf{p}_i)/m \\ &= (10 \text{ N toward Count Dooku} * 0.5 \text{ s} + 1 \text{ kg} * 0 \text{ m/s}) / 1 \text{ kg} \\ &= (5 \text{ N*s toward Count Dooku}) / 1 \text{ kg} \\ &= (5 \text{ kg*m/s}^2 * \text{s toward Count Dooku}) / 1 \text{ kg} \\ &= (5 \text{ kg*m/s toward Count Dooku}) / 1 \text{ kg} \\ &= \underline{5 \text{ m/s toward Count Dooku}}\end{aligned}$$

$$\begin{aligned}4.1) P &= F/A = 3 \text{ N} / 0.05 \text{ cm}^2 \\ &= \underline{60 \text{ N/cm}^2}\end{aligned}$$

$$\begin{aligned}4.2) P &= F/A = 3 \text{ N} / 0.001 \text{ cm}^2 \\ &= \underline{3,000 \text{ N/cm}^2} \quad \text{This is why sharpening knives makes them cut better!}\end{aligned}$$